Two-component fluid membranes near repulsive walls: Linearized hydrodynamics of equilibrium and nonequilibrium states

Sumithra Sankararaman* and Gautam I. Menon†
The Institute of Mathematical Sciences, C.I.T. Campus, Taramani, Chennai 600 113, India

P. B. Sunil Kumar‡
Department of Physics, Indian Institute of Technology Madras, Chennai 600 036, India
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We study the linearized hydrodynamics of a two-component fluid membrane near a repulsive wall, using a model that incorporates curvature-concentration coupling as well as hydrodynamic interactions. This model is a simplified version of a recently proposed one [J.-B. Manneville *et al.*, Phys. Rev. E **64**, 021908 (2001)] for nonequilibrium force centers embedded in fluid membranes, such as light-activated bacteriorhodopsin pumps incorporated in phospholipid egg phosphatidyl choline (EPC) bilayers. The pump-membrane system is modeled as an impermeable, two-component bilayer fluid membrane in the presence of an ambient solvent, in which one component, representing active pumps, is described in terms of force dipoles displaced with respect to the bilayer midpoint. We first discuss the case in which such pumps are rendered *inactive*, computing the mode structure in the bulk as well as the modification of hydrodynamic properties by the presence of a nearby wall. These results should apply, more generally, to equilibrium fluid membranes comprised of two components, in which the effects of curvature-concentration coupling are significant, above the threshold for phase separation. We then discuss the fluctuations and mode structure in the steady state of active two-component membranes near a repulsive wall. We find that proximity to the wall *smoothens* membrane height fluctuations in the stable regime, resulting in a logarithmic scaling of the roughness even for initially tensionless membranes. This explicitly nonequilibrium result is a consequence of the incorporation of curvature-concentration coupling in our hydrodynamic treatment. This result also indicates that earlier scaling arguments which obtained an increase in the roughness of active membranes near repulsive walls upon neglecting the role played by such couplings may need to be reevaluated.

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I. INTRODUCTION

Amphiphilic molecules, in polar solvents such as water, form symmetric bilayer membrane phases at sufficient concentration [1–4]. Such membranes are typically *fluid* in nature, with short-ranged positional order, although more complex forms of ordering are possible [5–7]. The equilibrium conformations of such single-component membranes are governed by the energy cost for bending the bilayer, provided the surface tension can be neglected, as is the case if the membrane is self-assembled [4,8,9]. Our understanding of the static and dynamic properties of such single-component membrane systems in equilibrium is now fairly detailed [3,10–17].

More complex multicomponent membranes have also been synthesized and studied [9,18–20]. Such membranes are especially important in the biological context, since cell membranes are often usefully idealized as a bilayer *fluid mosaic* comprised of over a hundred different types of lipid and protein constituents [21,22]. Biological membranes, however, often have additional complicating features: An underlying network of cross-linked proteins anchored to the bilayer and associated with the cell cytoskeleton typically lends such membranes a nonvanishing (although small) shear modulus [22]. Another aspect, specific to the biological context, is the presence of nonequilibrium driving forces: An important class of transmembrane proteins are the ''ion pumps,'' molecules which consume energy derived from adenosine triphosphate hydrolysis, electrochemical gradients, or light, and undergo conformational changes. Such pumps maintain electro-osmotic potential gradients across the cell membrane by controlling the flow of ions such as K^+ and Na^+. Since energy must be supplied externally, the driving of the pump is a process that occurs out of thermal equilibrium. Thus, the situation of active pumps diffusing in a fluid membrane matrix is an intrinsically nonequilibrium problem whose behavior represents a class of nonequilibrium steady states.

The appropriate statistical description of active or ''driven'' biological membrane systems has attracted recent experimental and theoretical attention [23]. It has been realized that nonequilibrium behavior may underlie aspects of biomembrane dynamics [24,25], previously attributed purely to equilibrium thermal fluctuations, such as the ''flicker'' phenomenon in erythrocytes [26]. Recent micropipette experiments on light-activated bacteriorhodopsin pumps incorporated in phospholipid (EPC) bilayers find that area fluctuations in such active membrane systems can be phenomenologically described in terms of an effective temperature, which exceeds the true physical temperature by up to a factor of 2 [27,28]. In these systems, the transition be-

*Email address: sumithra@imsc.ernet.in
†Email address: menon@imsc.ernet.in
‡Email address: sunil@physics.iitm.ac.in

tween active and inactive states can be easily (and reversibly) tuned, providing a remarkable window into the dynamics of steady states away from equilibrium [28–31].

Analytical calculations which deal with the complex nature of a typical biological membrane *in toto* are difficult, if not impossible. We may, however, hope to gain useful insight by working with simpler models. We idealize a typical biological membrane here as composed of principally *two* different types of molecular constituents, the lipids, which constitute the bulk of the bilayer membrane, and the proteins, which diffuse freely on the membrane surface. Such simple lipid–pump protein systems can be reconstituted and studied *in vitro*, as in the micropipette experiments referred to above. Our results should apply, most directly, to such experiments.

The question we address in this paper is the following: How are the fluctuations of such a two-component membrane [32], both in equilibrium and out of equilibrium, affected by the presence of a nearby repulsive wall? Fluctuations of a single-component membrane bounded on one side by a repulsive wall have been considered in earlier work [3,33]. More recently, a similar problem for active membranes has been studied in an influential paper by Prost and co-workers [29,30], who suggest that steady-state fluctuations of nonequilibrium membranes near repulsive walls can be *amplified* substantially relative to the equilibrium case. This amplification is studied using a scaling treatment which incorporates nonlinear effects. Our starting model differs from theirs in several respects, in particular, in the way we describe activity, in our incorporation of curvature-concentration coupling, as well as in our evaluation of the relative importance of permeative and hydrodynamic effects at the length scales accessed by typical experimental probes. We study the *linearized* hydrodynamics of such active pump-membrane systems in this paper, ignoring the role of nonlinear effects. We will describe the similarities as well as the differences in the results we obtain.

We answer the question posed above first through a calculation of the correlations and linearized mode structure in thermal equilibrium of a model impermeable two-component fluid membrane, incorporating hydrodynamic interactions. This amounts, in terms of the model for biological membranes discussed above, to assuming that the pump proteins are ''passive'' constituents of the membrane. While motivated here in a biological context, such models are also useful outside this specific context. Physics similar to that described in the case where the pumps are rendered passive should be generic to two-component systems composed of (i) a symmetric (lipid) constituent forming an up-down symmetric bilayer phase in isolation, and (ii) an asymmetric (protein) constituent, in which either the shape of the molecule or its location with respect to the bilayer favors a local spontaneous curvature for the bilayer. For concreteness, we will often use the terms lipid and protein when referring to constituents of types (i) and (ii), following our earlier discussion, but stress that the results obtained here in the *equilibrium* case are also applicable more generally.

For protein molecules which lack up-down symmetry, we may distinguish between molecules oriented parallel to the local normal and those which are oriented antiparallel to the normal. We label these proteins as + and − for convenience; the relatively small flip-flop rate for protein transfer across leaves of the bilayer ensures that these labels are conserved across short and intermediate time scales. The difference between the local densities of + and − proteins defines a ''signed'' protein density field ψ. To describe the incorporation of such asymmetric proteins into the lipid bilayer, we model the protein as a rigid rod of length $w^{\uparrow}+w^{\downarrow}$. The protein is taken to situate itself asymmetrically with respect to the bilayer midpoint; a section $w^{\uparrow}$ of the protein lies on one side of the bilayer midpoint while a section $w^{\downarrow}$ lies on the other side. This relatively simple model of the asymmetry is convenient for analytic computation.

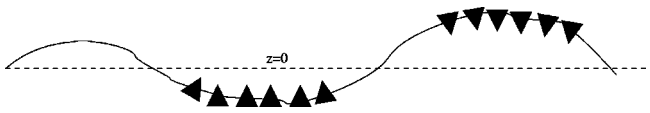

FIG. 1. A schematic diagram of the two kinds of proteins, represented as up and down triangles. These affect the local curvature of the membrane by bending it towards or away from the local normal. The bilayer midplane lies at $z=0$.

To lowest order, the two-component character of the membrane manifests itself via the existence of a curvature-concentration coupling—fluctuations in ψ influence the mean curvature in that region (see Fig. 1). We will work with balanced membranes for which $\langle\psi\rangle=0$. For inactive pumps, corresponding to the equilibrium case, the curvature-concentration coupling term in the free energy effectively shifts the bending rigidity of the membrane from κ to $\kappa_e=\kappa-(\kappa\bar{H})^2/\chi_0^{-1}$. Here $\bar{H}$ is the coefficient coupling curvature to concentration and χ_0^{-1} is an inverse compressibility [31].

In closed vesicular structures found in biological contexts, while the bilayer constituting the vesicle may be symmetric, chemical and physical environments within and outside the vesicle can differ considerably. Thus, a pump protein can, in principle, distinguish the side of the bilayer exposed to the inner volume from the side which is exposed to the outer volume. For a large enough vesicle, flat on the large length scales of relevance to a hydrodynamic calculation, the pump molecule can thus tell *up* from *down*, where *up* and *down* are defined arbitrarily with respect to the average position and orientation of the bilayer midpoint. As an extreme limit, we may consider the case in which the pump molecules preferentially insert always on the same (say up) side, so that a section $w^{\uparrow}$ of the protein molecule always lies *above* the bilayer midpoint while a section $w^{\downarrow}$ lies below.

A recent study of active membranes works with this model of the architecture of a membrane-pump system, modeling activity in terms of dipole forces associated with the ends of the pump, as we detail below [28]. The associated forces may point either outward at the ends (up pumps) or inward at the ends (down pumps); see Fig. 2. In this model, up and down pumps are physically distinguishable. The main results presented here are appropriate to this model.

Note that this distinction between ''above'' and ''below'' is artificial for strictly symmetric bilayer membranes, in which pump proteins may insert on either leaf of the bilayer

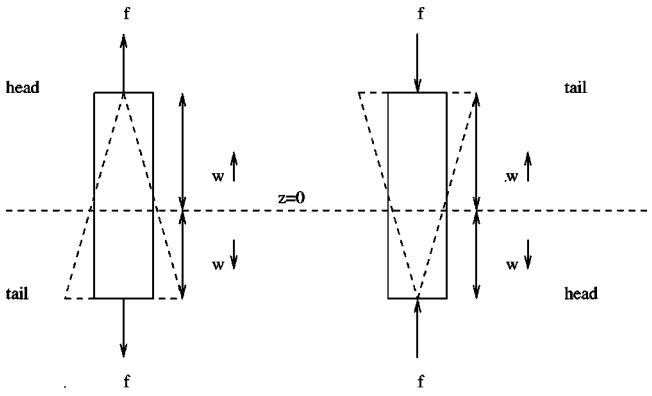

FIG. 2. The asymmetric dipole model used to describe the activity of the proteins. The force centers are points at distances $w^{\uparrow}$ and $w^{\downarrow}$ from the bilayer midpoint. The centers of mass of the pumps are displaced above the bilayer midpoint. The superposed up and down triangles indicate the underlying shape asymmetry of the two different kinds of pumps.

with equal probability. In the Appendix, we discuss results for an alternative model, in which the protein attachment respects the up-down symmetry of the bilayer; the center of mass of the protein can be displaced either above or below the bilayer midplane as shown in Fig. 3. The head and tail distances from the bilayer midpoint are fixed at w_l and w_s, respectively. In this model, up pumps and down pumps are physically indistinguishable; their nomenclature follows from the side of the bilayer on which they are inserted.

We will present results for *nonequilibrium* membranes in the bulk, in which case the pumps are active, and for the case in which fluctuations of such an active membrane are bounded on one side by a repulsive wall. This study complements results by Prost, Manneville, and Bruinsma for a model in which curvature-concentration coupling is ignored, an effect explicitly incorporated in later work [31]. In the active case, the pumps exert forces on the membrane and the surrounding fluid. Since these forces are internal forces, they must cancel when integrated over the size of the pump [34].

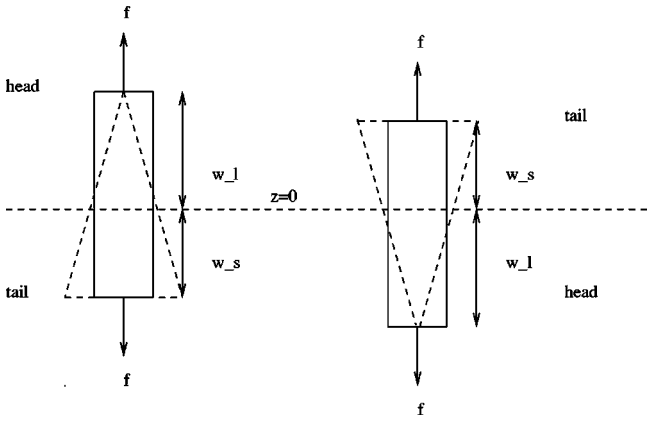

FIG. 3. The asymmetric dipole model of the pumps described in the Appendix. The centers of mass of the pumps are displaced above or below the bilayer midpoint. The distance of the furthest force center from $z=0$ is w_l and the distance of the nearest force center from $z=0$ is w_s.

Following Manneville *et al.* in Ref. [28], we consider the simplest model for pumps consistent with this requirement: the pumps are taken to be dipole force centers with positive and negative force centers located asymmetrically with respect to the midpoint of the bilayer.

Our other major assumptions are the following: We assume that our observations are conducted on scales such that the membrane may be considered to be *impermeable* to the solvent in which it is embedded. This assumption follows from the work of Manneville *et al.*, who observe that for active terms, permeative effects can be neglected *vis-à-vis* hydrodynamic ones over a substantial range of length scales upto microscopic ones. We derive our results under the assumption that the active membrane is impermeable and extend this assumption to the case when the membrane is in the passive state, to smoothly interpolate between results in the two regimes. (In the Rouse or free-draining limit, where the membrane dynamics decouples from that of the fluid, the membrane is implicitly permeable.) We take the solvent to be incompressible and work at a small Reynolds number, neglecting the inertia of the fluid. We use linearized equations of motion for the hydrodynamic velocity field, corresponding to the Stokes limit of the Navier-Stokes equations.

We now summarize our principal results. For *equilibrium*, impermeable, two-component membranes bounded on one side by a wall placed at a distance d from the membrane plane, we obtain the following modified mode structure to leading order:

$$\omega_1 \approx \mathcal{G}(\sigma k_\perp^2 + \kappa_e k_\perp^4),\ \omega_2 \approx D k_\perp^2 . \tag{1.1}$$

In our notation, D is the diffusion coefficient of the proteins on the membrane, σ the surface tension of the membrane, κ the bending rigidity of the membrane, and η the viscosity of the fluid. Here,

$$\mathcal{G}(k_\perp, k_\perp d) \sim \begin{cases} d^3 k_\perp^2 / 12\eta & \text{if } k_\perp \ll 1/d \\ 1/4\eta k_\perp & \text{if } k_\perp \gg 1/d \end{cases} \tag{1.2}$$

is a ''crossover function,'' which describes how membrane fluctuations interpolate between a regime where physical interactions between the membrane and the wall cannot be neglected, and a regime where such fluctuations do not feel the presence of the wall [33]. In the $k_\perp \gg 1/d$ limit, the above result reduces to a derivable one in the bulk case, as is physically sensible.

To study *nonequilibrium* membranes we model active pumps, following Manneville *et al.*, as asymmetric force dipoles, with force centers located at distances $w^{\uparrow}$ above the bilayer midpoint and $w^{\downarrow}$ below the bilayer midpoint. These distances are, in general, unequal. The magnitude of the force exerted at each of these force centers is f. The pumps diffuse on the membrane and are characterized by a mobility Λ.

In the presence of a wall, for nonequilibrium membranes, it has been argued that static height fluctuations are *amplified* and an interesting speculation is that such an amplification

may, in fact, speed up the process of membrane fusion when two *nonequilibrium* membranes are brought close together [27]. When the wall is at a distance d from the membrane, our calculation yields (for a tensionless membrane)

$$\omega_1 \approx \frac{f\mathcal{G}^{aw}\kappa\overline{H}}{\chi_0^{-1}}k_\perp^2 + \mathcal{G}\kappa_e k_\perp^4,$$

$$\omega_2 \approx Dk_\perp^2 + \frac{\kappa\overline{H}(\kappa\overline{H}\mathcal{G}k_\perp^2 - f\mathcal{G}^{aw})}{\chi_0^{-1}}k_\perp^2, \tag{1.3}$$

where $\mathcal{G}^{aw}$ is the crossover function,

$$\mathcal{G}^{aw}(k_\perp, d, w^\uparrow, w^\downarrow) \sim \begin{cases} dw^{\downarrow 2}k_\perp^2/4\eta & \text{if } k_\perp \ll 1/d \\ \Omega(k_\perp, w^\uparrow, w^\downarrow)/4\eta k_\perp & \text{if } k_\perp \gg 1/d. \end{cases} \tag{1.4}$$

$\Omega(k_\perp, w^\uparrow, w^\downarrow)$ is proportional to the degree of asymmetry $(w^\downarrow - w^\uparrow)$. In a regime where fluctuations feel the wall, nonequilibrium effects do not depend on the asymmetry factor but only on $w^\downarrow$, as seen above. Here again, curvature-concentration coupling plays a crucial role in the manifestation of nonequilibrium effects in the linearized mode structure.

We find that in the presence of the wall, the first term in ω_1 is the most important one since it is of order $O(k_\perp^4)$. The existence of this mode depends crucially on the presence of both activity and curvature-concentration coupling. It will be shown later in this paper that this mode causes active membranes near walls to relax faster than their equilibrium counterparts and causes a smoothening of membrane fluctuations; in the presence of a wall, the roughness of *tensionless*, active membranes averaged over a patch of size $L \times L$, $\langle h^2(L)\rangle$, scales as $\ln(L)$ instead of L^2. Thus the wall acts to *smoothen* active membrane fluctuations in a tensionless membrane by inducing an apparent surface tension. This result is a purely nonequilibrium one, since the apparent surface tension vanishes when the strength of active forces is set to zero. It also relies crucially on the existence of a curvature-concentration coupling, since the apparent surface tension also vanishes when the curvature and concentration fields are decoupled. We show that this effect can be understood in terms of the dynamics of the fast relaxing ψ field, which drives the height field $h(\mathbf{r}_\perp, t)$ *via* activity and curvature-concentration coupling.

The outline of this paper is as follows: Section II describes our basic coarse-grained free energy and reviews the calculation of the static properties of a two-component membrane with curvature-concentration coupling. In Sec. III A, the free-draining approximation (Rouse model) is used to discuss the dynamics. In Sec. III B, we incorporate the effects of the hydrodynamic velocity field of the solvent. Section IV describes the mode structure in the presence of a wall for the equilibrium problem. In Sec. V, we discuss the mode structure and correlation functions of active two-component membranes. Section VI discusses how these properties are modified by the presence of a wall. In Sec. VII, we summarize the conclusions of this paper. The Appendix discusses the details of an alternative model for active two-component membranes, with up-down symmetry.

II. STATICS

We model the membrane as an infinitesimally thin two-dimensional surface embedded in three-dimensional space. Points on this surface are specified by a three-dimensional vector $\mathbf{R}(\tilde{u})$, with components $R_i(\tilde{u})$, $i=1,\ldots,3$. We use the Monge gauge, valid for nearly flat surfaces, in which the surface is specified by its height above a two-dimensional flat plane. This plane is parametrized via Euclidean coordinates, i.e., $\tilde{u}=(x,y)\equiv\mathbf{r}_\perp$; $\mathbf{R}(\mathbf{r}_\perp)=(x,y,h(x,y))$. In this gauge, the curvature is $H=-\nabla^2 h$, to lowest order.

The number densities of the lipid molecules, + proteins, and − proteins are specified through fields ρ_ℓ, $\rho^+(\tilde{u},t)$, and $\rho^-(\tilde{u},t)$, respectively. These are the physical densities of the lipids and proteins on the membrane. The projections of these densities on the x-y plane are obtained by multiplying the physical density fields by the metric factor $\sqrt{1+(\nabla h)^2}$. To lowest order in gradients of h, physical densities and projected densities coincide. The scalar fields $\phi(\tilde{u},t)$ and $\psi(\tilde{u},t)$ are defined by $\phi(\tilde{u},t)=\rho^+(\tilde{u},t)+\rho^-(\tilde{u},t)$ and $\psi(\tilde{u},t)=\rho^+(\tilde{u},t)-\rho^-(\tilde{u},t)$.

The free energy for the membrane-protein system consists of a part arising from protein and lipid densities (the matter part) and another which arises as a consequence of membrane fluctuations about the flat state. Given equilibrium concentrations of the lipid and protein (ρ_0), $\langle\phi(\tilde{u},t)\rangle=\langle\rho^+(\tilde{u},t)+\rho^-(\tilde{u},t)\rangle=\phi_0$, $\langle\psi(\tilde{u},t)\rangle=\langle\rho^+(\tilde{u},t)-\rho^-(\tilde{u},t)\rangle=\psi_0$ and fixing $\rho\simeq\rho_0$ and $\phi\simeq\phi_0$, the appropriate gauge-invariant free energy of the membrane with asymmetric proteins is

$$F=\int d^2\tilde{u}\sqrt{g}\left[\sigma+\frac{1}{2}\chi_0^{-1}(\psi-\psi_0)^2\right] + \frac{1}{2}\kappa\int d^2\tilde{u}\sqrt{g}(H-H_0)^2, \tag{2.1}$$

where σ is the surface tension and κ is the bending rigidity. To lowest order we take $H_0=\overline{H}\psi$. To leading order in gradients of h, we have

$$F=\frac{1}{2}\int d^2\mathbf{r}_\perp[\kappa\{\nabla^2 h(\mathbf{r}_\perp,t)\}^2+\sigma\{\nabla h(\mathbf{r}_\perp,t)\}^2 - 2\overline{H}\kappa\psi(\mathbf{r}_\perp,t)\nabla^2 h(\mathbf{r}_\perp,t)+\chi_0^{-1}\psi(\mathbf{r}_\perp,t)^2], \tag{2.2}$$

yielding a local force density of $\mathbf{f}(\mathbf{r}_\perp)=-\delta F/\delta h|_\psi$. The local force density is evaluated at constant density [35].

The partition function is [36]

$$Z=\int \mathcal{D}[h]\mathcal{D}[\psi]e^{-\beta F[h,\psi]}, \tag{2.3}$$

which yields, on integrating out the ψ field, an effective free energy for the height field $F_{eff}[h]$,

$$F_{eff}[h]=\frac{1}{2}\sum_{k_\perp}(\sigma k_\perp^2+\kappa k_\perp^4)|h(k_\perp)|^2-\frac{(\kappa\bar{H})^2}{\chi_0^{-1}}k_\perp^4|h(k_\perp)|^2. \tag{2.4}$$

The equipartition theorem yields the equilibrium correlation function for the height field fluctuations (about the equilibrium, flat, ''mixed'' phase) as

$$\langle h^*(\mathbf{k}_\perp)h(\mathbf{k}_\perp)\rangle=\frac{k_BT}{(\sigma k_\perp^2+\kappa_e k_\perp^4)}, \tag{2.5}$$

where $\kappa_e=\kappa-(\kappa\bar{H})^2/\chi_0^{-1}$. The asymmetric components act to reduce the rigidity modulus from κ to κ_e as a consequence of curvature-concentration coupling [31], an effect easily understood on physical grounds. We assume that $\bar{H}$ is small enough to keep κ_e positive, in which case the membrane is linearly stable. Instabilities arising at equilibrium due to large values of $\bar{H}$ have been discussed in [37].

III. MODE STRUCTURE IN THE BULK

We now study the dynamics of a two-component membrane with curvature-concentration coupling and suspended in a solvent. We will first present results in the Rouse model limit in which the background fluid drains freely through the membrane, and effects due to the hydrodynamics of the background fluid velocity field can be ignored [38].

A. Rouse model

In this model the fluid velocity field decouples from the membrane height field and ψ. The membrane is implicitly permeable and the height field fluctuations are governed by a balance between the elastic forces and local (permeative) friction. An equation of the form

$$\partial_t h(\mathbf{r}_\perp,t)=-\Gamma\frac{\delta F}{\delta h}+f_m, \tag{3.1}$$

follows, where Γ is a kinetic coefficient and f_m is a thermal equilibrium noise whose statistics ensures that the static height-height correlation function is Eq. (2.5).

The proteins diffuse freely on the surface of the membrane. Since ψ is locally conserved, it obeys an equation of continuity of the form

$$\partial_t\psi(\mathbf{r}_\perp,t)=\Lambda\nabla^2\frac{\delta F}{\delta\psi}+\boldsymbol{\nabla}\cdot\mathbf{f}_\psi, \tag{3.2}$$

with $\Lambda=D/\chi_0^{-1}$, D is the diffusion coefficient of the proteins on the membrane. A contribution due to the in-plane current of the pumps has been dropped; this is typically small and can be neglected to linear order. This term contributes if we go beyond the strictly linear treatment by replacing ψ^2 by $\langle\psi^2\rangle$, yielding a convective term which gives rise to waves whose speed is independent of the wave vector [23,31]. The last term is a conserving Gaussian noise with $\langle\mathbf{f}_\psi(\mathbf{r}_\perp,\mathbf{t})\rangle=\mathbf{0}$ and correlations $\langle f_{\psi i}(\mathbf{r}_\perp,t)f_{\psi j}(\mathbf{r}'_\perp,t')\rangle=2\Lambda k_BT\delta_{ij}\delta(\mathbf{r}_\perp-\mathbf{r}'_\perp)\delta(t-t')$.

Equations (3.1) and (3.2) can be written as

$$\partial_t\begin{pmatrix}h(\mathbf{k}_\perp,\omega)\\ \psi(\mathbf{k}_\perp,\omega)\end{pmatrix}=-\begin{pmatrix}\Gamma(\sigma k_\perp^2+\kappa k_\perp^4) & \Gamma\kappa\bar{H}k_\perp^2\\ \Lambda\kappa\bar{H}k_\perp^4 & \Lambda\chi_0^{-1}k_\perp^2\end{pmatrix}\times\begin{pmatrix}h(\mathbf{k}_\perp,\omega)\\ \psi(\mathbf{k}_\perp,\omega)\end{pmatrix}+\begin{pmatrix}f_m\\ i\mathbf{k}_\perp\cdot\mathbf{f}_\psi\end{pmatrix}. \tag{3.3}$$

Solving these coupled equations yields

$$\langle h^*(\mathbf{k}_\perp,\omega)h(\mathbf{k}_\perp,\omega)\rangle=\frac{\langle f_m f_m\rangle(\omega^2+\tau_\psi^{-12})+2(\Gamma\kappa\bar{H})^2\Lambda k_BTk_\perp^6}{[-\omega^2+\tau_h^{-1}\tau_\psi^{-1}-\Lambda\Gamma(\kappa\bar{H})^2k_\perp^6]^2+\omega^2(\tau_\psi^{-1}+\tau_h^{-1})^2}, \tag{3.4}$$

where $\tau_h^{-1}=\Gamma(\sigma k_\perp^2+\kappa k_\perp^4)$ and $\tau_\psi^{-1}=\Lambda\chi_0^{-1}k_\perp^2$.

On integrating over ω, this correlation function should reduce to the same form as Eq. (2.5). This constrains the equilibrium thermal noise correlations to be the following:

$$\langle f_m(\mathbf{r}_\perp,t)f_m(\mathbf{r}'_\perp,t')\rangle=2\Gamma k_BT\delta(\mathbf{r}_\perp-\mathbf{r}'_\perp)\delta(t-t'). \tag{3.5}$$

In addition, we impose $\langle f_m(\mathbf{r}_\perp,t)\rangle=0$.

To obtain the mode structure, we drop the noise terms in Eq. (3.3) and find the eigenvalues of the matrix. This yields

$$\omega_1,\omega_2=\frac{1}{2}[\Gamma(\sigma k_\perp^2+\kappa k_\perp^4)+Dk_\perp^2]\pm\frac{1}{2}\{\sqrt{[\Gamma(\sigma k_\perp^2+\kappa k_\perp^4)+Dk_\perp^2]^2-4D\Gamma k_\perp^2(\sigma k_\perp^2+\kappa_e k_\perp^4)}\}.$$

In the absence of curvature-concentration coupling these modes decouple and are $\omega_1, \omega_2 = \Gamma(\sigma k_\perp^2 + \kappa k_\perp^4)$, $Dk_\perp^2$, the dispersive modes of a tense membrane [39]. For tensionless membranes, the presence of curvature-concentration coupling modifies the mode structure to

$$\omega_1 \approx \Gamma \kappa_e k_\perp^4, \omega_2 \approx Dk_\perp^2 + \frac{(\kappa \bar{H})^2}{\chi_0^{-1}} \Gamma k_\perp^4. \tag{3.6}$$

B. Zimm model

Next, we examine the effect of solvent hydrodynamics on the dynamical correlation functions of the protein density field and the membrane height field [38]. Since no external forces act on the (fluid+membrane+proteins) system, momentum conservation requires [40,41]

$$\partial_t g_i = -\nabla_j \pi_{ij}, \tag{3.7}$$

where $\mathbf{g}$ is the momentum density and π_{ij} is the stress tensor. The stress tensor takes the form $\pi_{ij} = P\delta_{ij} + \rho v_i v_j - \eta(\nabla_i v_j + \nabla_j v_i) + \mathcal{S}_{ij}$, where $\mathbf{v}(\mathbf{r},t)$ is the fluid velocity, $P(\mathbf{r},t)$ is the pressure field, and $\mathcal{S}_{ij}$ describes the contribution to the stress tensor arising from membrane conformations (the ∇^2 in the above equation is evaluated in three dimensions). In the Stokes limit, the nonlinear term can be dropped. At low Reynolds number, we neglect the inertial term on the left-hand side of Eq. (3.7) leading to $\nabla_j \pi_{ij} = 0$, and thus the equation governing the hydrodynamic velocity field,

$$\eta \nabla^2 \mathbf{v}(\mathbf{r},t) - \nabla P(\mathbf{r},t) - \mathbf{F}(\mathbf{r},t) + \mathbf{f}_h(\mathbf{r},t) = 0. \tag{3.8}$$

$\mathbf{F}(\mathbf{r},t)$ is the force acting on the fluid due to membrane stresses $\mathcal{S}_{ij}$ and $\mathbf{f}_h(\mathbf{r},t)$ is the equilibrium thermal noise present in the bulk of the fluid. This noise obeys $\langle \mathbf{f}_h(\mathbf{r},t) \rangle = 0$ [28,29] and is related to the viscosity via the fluctuation-dissipation theorem,

$$\langle f_{hi}(\mathbf{r},t) f_{hj}(\mathbf{r}',t') \rangle = 2k_B T \eta \{ -\delta_{ij} \nabla^2 + \partial_i \partial_j \} \times \delta(\mathbf{r}-\mathbf{r}') \delta(t-t'), \tag{3.9}$$

where (i,j) index the components of the force. Equations (3.8) are to be solved together with the incompressibility condition

$$\nabla \cdot \mathbf{v}(\mathbf{r},t) = 0 \tag{3.10}$$

to obtain the mode structure of an impermeable two-component membrane in the presence of an incompressible solvent [42].

We employ a useful convention due to Seifert in which a basis spanned by the z component and the longitudinal and transverse *in-plane* components of the velocity is used [3]. (This convention will be useful further on in this paper when we discuss membrane fluctuations in the presence of a confining wall.) We solve the above equations to obtain the transverse (t), longitudinal (l), and z components of the fluid velocity in terms of the t, l, and z components of the force $\mathbf{F}$ acting on the fluid [43]. The t and l components of the velocity lie in the plane of the membrane and are perpendicular and parallel to $\mathbf{k}_\perp$, the wave vector describing fluctuations of the membrane. Some algebra yields these components as [3]

$$v_z(\mathbf{k}_\perp, z) = \frac{1}{4\eta k_\perp} \int_{-\infty}^{\infty} dz' e^{-k_\perp |z'-z|} [(1 + k_\perp |z-z'|) \times F_z(\mathbf{k}_\perp, z') + ik_\perp (z'-z) F_l(\mathbf{k}_\perp, z')],$$

$$v_l(\mathbf{k}_\perp, z) = \frac{1}{4\eta k_\perp} \int_{-\infty}^{\infty} dz' e^{-k_\perp |z'-z|} [(1 - k_\perp |z-z'|) \times F_l(\mathbf{k}_\perp, z') + ik_\perp (z'-z) F_z(\mathbf{k}_\perp, z')], \tag{3.11}$$

$$v_t(\mathbf{k}_\perp, z) = \frac{1}{4\eta k_\perp} \int_{-\infty}^{\infty} dz' e^{-k_\perp |z'-z|} 2F_t(\mathbf{k}_\perp, z'). \tag{3.12}$$

Note that the transverse components can be ignored since they do not couple to the other components. We can now calculate $v_z(\mathbf{k}_\perp, 0)$ by supplementing the above equations with boundary conditions.

In the absence of permeation the membrane is advected by the fluid. The velocity of any point on the membrane is the z component of the velocity of the fluid at that point [28]. The equation of motion for the height field of an impermeable membrane is therefore

$$\partial_t h(\mathbf{r}_\perp, t) = v_z(\mathbf{r}_\perp, 0, t), \tag{3.13}$$

where $v_z(\mathbf{r}_\perp, 0, t)$ is the velocity of the surrounding fluid at the mean position of the membrane. The force acting on the fluid is of the form

$$\mathbf{F}(\mathbf{k}_\perp, z) = \delta(z) \left[-\frac{\delta F}{\delta h} \hat{\mathbf{z}} + F_l^m \hat{\mathbf{k}}_\perp \right], \tag{3.14}$$

where F_l^m is the longitudinal component of the force acting on the fluid due to the membrane. The elastic force density due to the membrane is $-\delta F / \delta h \hat{\mathbf{z}}$. We solve for $v_z(\mathbf{k}_\perp, 0)$ using Eqs. (3.11), together with the boundary condition that the membrane is incompressible [35], i.e., the in-plane divergence of the velocity field vanishes at the membrane. This implies that $v_l(\mathbf{k}_\perp, 0) = 0$. This boundary condition gives $F_l^m = 0$, yielding

$$v_z(\mathbf{k}_\perp, 0) = -\frac{1}{4\eta k_\perp} \frac{\delta F}{\delta h}. \tag{3.15}$$

The bulk thermal noise present in the fluid also contributes to $v_z(\mathbf{k}_\perp, 0)$. This contribution is denoted by f_z, with

$$f_z(\mathbf{k}_\perp, 0) = \int \frac{dk_z}{2\pi\eta k^2} \left(\delta_{jz} - \frac{k_j k_z}{k^2} \right) [f_h(\mathbf{k}, t)]_j. \tag{3.16}$$

We can use the noise correlation function given in Eq. (3.9) to show that the noise f_z has zero mean and that its variance in Fourier space is $2k_BT/4\eta k_\perp$.

Finally, Eq. (3.13) becomes

$$\partial_t h = -\frac{1}{4\eta k_\perp}\frac{\delta F}{\delta h} + f_z(\mathbf{k}_\perp,0,t). \tag{3.17}$$

The density difference field obeys Eq. (3.2). Equations (3.13) and (3.2) can then be written as

$$\partial_t \begin{pmatrix} h(\mathbf{k}_\perp,\omega) \\ \psi(\mathbf{k}_\perp,\omega) \end{pmatrix} = -\begin{pmatrix} \dfrac{\sigma k_\perp^2+\kappa k_\perp^4}{4\eta k_\perp} & \dfrac{\kappa\bar{H}k_\perp^2}{4\eta k_\perp} \\ \Lambda\kappa\bar{H}k_\perp^4 & \Lambda\chi_0^{-1}k_\perp^2 \end{pmatrix}\begin{pmatrix} h(\mathbf{k}_\perp,\omega) \\ \psi(\mathbf{k}_\perp,\omega) \end{pmatrix} + \begin{pmatrix} f_z(\mathbf{k}_\perp,\omega) \\ i\mathbf{k}_\perp\cdot\mathbf{f}_\psi \end{pmatrix}. \tag{3.18}$$

Solving these equations gives

$$\langle h^*(\mathbf{k}_\perp,\omega)h(\mathbf{k}_\perp,\omega)\rangle = \frac{(2k_BT/4\eta k_\perp)(\omega^2+\tau_\psi^{-1^2})+[k_BT\Lambda(\kappa\bar{H})^2/8\eta^2]k_\perp^4}{\{-\omega^2+\tau_h^{-1}\tau_\psi^{-1}-[\Lambda(\kappa\bar{H})^2/4\eta]k_\perp^5\}^2+\omega^2(\tau_\psi^{-1}+\tau_h^{-1})^2}. \tag{3.19}$$

The parameters appearing in the above equations are

$$\tau_h^{-1}=\frac{1}{4\eta k_\perp}(\sigma k_\perp^2+\kappa k_\perp^4), \quad \tau_\psi^{-1}=\Lambda\chi_0^{-1}k_\perp^2. \tag{3.20}$$

Integrating this result over ω recovers for us the equal time correlation function for height fluctuations about the equilibrium, flat, mixed phase,

$$\langle h^*(\mathbf{k}_\perp,t)h(\mathbf{k}_\perp,t)\rangle = \frac{k_BT}{(\sigma k_\perp^2+\kappa_e k_\perp^4)}, \tag{3.21}$$

as expected. The roughness of the height field at macroscopic length scales L, defined through $\langle h^2(L)\rangle = \int_{1/L}^{1/a} d^2k_\perp \langle h(k_\perp)h^*(k_\perp)\rangle$, scales as

$$\langle h^2(L)\rangle = \begin{cases} \dfrac{k_BT}{\kappa_e}L^2 & \text{if } \sigma=0 \\ \dfrac{k_BT}{\sigma}\ln(L/a) & \text{if } \sigma\neq 0, \end{cases} \tag{3.22}$$

where a is a microscopic cutoff length.

The mode structure is then

$$\omega_1,\omega_2=\frac{1}{8\eta}(\sigma k_\perp+\kappa k_\perp^3+4\eta D k_\perp^2)\pm\frac{1}{8\eta}[\sqrt{(\sigma k_\perp+\kappa k_\perp^3+4\eta D k_\perp^2)^2-16\eta D(\kappa_e k_\perp^5+\sigma k_\perp^3)}].$$

In the absence of curvature-concentration coupling, $\omega_1,\omega_2=(\sigma k_\perp+\kappa k_\perp^3)/4\eta, Dk_\perp^2$, i.e., the decoupled modes of the membrane height field and the protein concentration field in the presence of hydrodynamic correlations. For tensionless membranes with nonzero curvature-concentration coupling, the modes are

$$\omega_1\approx\frac{\kappa_e}{4\eta}k_\perp^3, \quad \omega_2\approx Dk_\perp^2+\frac{(\kappa\bar{H})^2}{4\eta\chi_0^{-1}}k_\perp^3. \tag{3.23}$$

IV. MODE STRUCTURE IN THE PRESENCE OF A WALL

Consider a fluid bilayer membrane with shape asymmetric constituents, suspended in a fluid bounded on one side by a wall. Our model for the membrane-pump system follows that of Manneville *et al.* In our coordinates, the membrane fluctuates about the $z=d$ plane and the wall is at $z=0$. The membrane is kept at distance d from the wall by a repulsive interaction with the wall and a suitable external pressure which prevents it from floating away into the bulk.

The relaxation rate of an equilibrium membrane is reduced near a wall; in tandem, the equilibrium noise fluctuations are required to have a reduced amplitude as a consequence of the fluctuation-dissipation theorem [29]. On short length scales, however, membrane fluctuations do not see the wall, and the height-height correlation function is identical to that in the bulk. This can be used to determine the form of thermal correlation functions in the presence of the wall.

We work in the linear regime where $\langle h^2(L_c)\rangle\approx d^2$, defining a collision length L_c. Beyond this length the membrane senses the presence of the wall, and the nonlinearities neglected in our treatment become important; $1/L_c$ is the smallest wave vector above which the linearization condition $|h(\mathbf{r}_\perp,t)|\ll d$ is valid. From Eq. (3.22) we can estimate the collision length of an equilibrium membrane near a wall to be

$$L_c(d)=\begin{cases}\sqrt{\dfrac{\kappa_e}{k_BT}}d & \text{if } \sigma=0\\ a\exp(\sigma d^2/k_BT) & \text{if } \sigma\neq 0.\end{cases} \tag{4.1}$$

We now solve Eq. (3.8) to obtain the velocity of the fluid at the position of the membrane. The membrane and wall are introduced in Eq. (3.8) as external forces acting on the fluid, imposing the required boundary conditions. The incompressibility of the fluid is used to solve for the components of the velocity in terms of the components of the external force.

We express the components of the velocity in the ''mixed'' basis of Seifert [3] as before. The force exerted by the membrane and wall on the fluid is of the form [3]

$$\mathbf{F}(\mathbf{k}_\perp,z)=\delta(z-d)\left[-\frac{\delta F}{\delta h}\hat{\mathbf{z}}+F_l^m\hat{\mathbf{k}_\perp}\right]+\delta(z)[F_z^w\hat{\mathbf{z}}+F_l^w\hat{\mathbf{k}_\perp}, \tag{4.2}$$

where F_z^w and F_l^w are the z and longitudinal components of the force exerted by the wall on the fluid at $z=0$. The quantities F_l^m, F_z^w, and F_l^w can be evaluated using the boundary conditions: all components of the velocity should vanish at the wall (no-slip boundary condition) [41]. This imposes

$$v_z(\mathbf{k}_\perp,0,t)=v_l(\mathbf{k}_\perp,0,t)=v_t(\mathbf{k}_\perp,0,t)=0. \tag{4.3}$$

These conditions, together with membrane incompressibility $[v_l(\mathbf{k}_\perp,d,t)=0]$, provide the boundary conditions for the velocity field of the fluid at the membrane. This gives $v_z(k_\perp,d,t)=-\mathcal{G}\delta F/\delta h$. The equation of motion for the height field is then obtained from Eq. (3.13) [33],

$$\partial_t h(\mathbf{k}_\perp,t)=-\mathcal{G}\frac{\delta F}{\delta h}+\tilde{f}_z(\mathbf{k}_\perp,d,t), \tag{4.4}$$

where

$$\mathcal{G}(k_\perp,k_\perp d)=\frac{1}{4\eta k_\perp}\left(\frac{2[\sinh^2(k_\perp d)-(k_\perp d)^2]}{\sinh^2(k_\perp d)-(k_\perp d)^2+k_\perp d+\sinh(k_\perp d)\cosh(k_\perp d)}\right)\simeq\begin{cases}d^3k_\perp^2/12\eta & \text{if } k_\perp\ll 1/d\\ 1/4\eta k_\perp & \text{if } k_\perp\gg 1/d.\end{cases} \tag{4.5}$$

We note that the term for the condition $k_\perp\ll 1/d$ is incorrect for arbitrarily small wave vectors. The relevant cutoff is the inverse of the collision length L_c since linearized calculations are not valid at wave vectors smaller than this value.

The contribution to the velocity field at the membrane due to the noise in the fluid in the presence of the wall is $\tilde{f}_z$. Following the procedure used in the derivation of Eq. (3.18), we get

$$\partial_t\begin{pmatrix}h(\mathbf{k}_\perp,\omega)\\ \psi(\mathbf{k}_\perp,\omega)\end{pmatrix}=-\begin{pmatrix}\mathcal{G}(\sigma k_\perp^2+\kappa k_\perp^4) & \mathcal{G}(\kappa\bar{H}k_\perp^2)\\ \Lambda\kappa\bar{H}k_\perp^4 & \Lambda\chi_0^{-1}k_\perp^2\end{pmatrix}\times\begin{pmatrix}h(\mathbf{k}_\perp,\omega)\\ \psi(\mathbf{k}_\perp,\omega)\end{pmatrix}+\begin{pmatrix}\tilde{f}_z(\mathbf{k}_\perp,\omega)\\ i\mathbf{k}_\perp\cdot\mathbf{f}_\psi\end{pmatrix}. \tag{4.6}$$

This matrix equation is solved to yield

$$\langle h^*(\mathbf{k}_\perp,\omega)h(\mathbf{k}_\perp,\omega)\rangle=\frac{\langle\tilde{f}_z\tilde{f}_z\rangle(\omega^2+\tau_\psi^{-12})+2k_BT\Lambda(\mathcal{G}\kappa\bar{H})^2k_\perp^6}{[-\omega^2+\tau_h^{-1w}\tau_\psi^{-1}-\mathcal{G}\Lambda(\kappa\bar{H})^2k_\perp^6]^2+\omega^2(\tau_\psi^{-1}+\tau_h^{-1w})^2}, \tag{4.7}$$

where $\tau_h^{-1w}=\mathcal{G}(\sigma k_\perp^2+\kappa k_\perp^4)$.

The hydrodynamics of the solvent cannot change equilibrium correlation functions. Thus, demanding that equal time correlations of the h field are the same as in Eq. (3.21), we obtain the noise correlation function of $\tilde{f}_z$ as

$$\langle\tilde{f}_z(\mathbf{k}_\perp,t)\rangle=0, \tag{4.8}$$

$$\langle\tilde{f}_z(\mathbf{k}_\perp,t)\tilde{f}_z(\mathbf{k}'_\perp,t')\rangle=2k_BT\mathcal{G}(k_\perp,k_\perp d)\,\delta(\mathbf{k}_\perp+\mathbf{k}'_\perp)\,\delta(t-t'). \tag{4.9}$$

Note that this reduces to the noise correlation function for the equilibrium bulk noise when $k_\perp\gg 1/d$, i.e., when the membrane fluctuations are not significantly affected by the wall (see Sec. III).

We note here that the equation of motion of the height field can be derived in the limit $1/L_c\ll k_\perp\ll 1/d$ by directly imposing impermeability as in [29]. Since the membrane is impermeable, the height field is conserved and obeys a continuity equation $\partial_t h=-\boldsymbol{\nabla}_\perp\cdot j_h$. The height ''current'' can be calculated from the lubrication approximation as $j_h=(-d^3/12\eta)\boldsymbol{\nabla}_\perp P$, where P is the pressure due to the elastic forces and the noise. Using the appropriate expressions for these terms, we recover the equation for the height [Eq. (4.4)] in the close to the wall limit.

The eigenmodes follow from Eq. (4.6),

$$\omega_1,\omega_2=\frac{1}{2}(Dk_\perp^2+\sigma\mathcal{G}k_\perp^2\pm\kappa\mathcal{G}k_\perp^4)+\frac{1}{2}\times[\sqrt{(\sigma\mathcal{G}k_\perp^2+Dk_\perp^2+\kappa\mathcal{G}k_\perp^4)^2-4\mathcal{G}D(\sigma+\kappa_e k_\perp^2)k_\perp^4}].$$

In the absence of curvature-concentration coupling the modes corresponding to the hydrodynamic relaxation of a two-component membrane near a wall are recovered: $\omega_1,\omega_2=\mathcal{G}(\sigma k_\perp^2+\kappa k_\perp^4),Dk_\perp^2$. For membranes with curvature-concentration coupling, the modes are modified to

$$\omega_1\approx\mathcal{G}(\sigma k_\perp^2+\kappa_e k_\perp^4),\quad \omega_2\approx Dk_\perp^2+\mathcal{G}\frac{(\kappa\bar{H})^2}{\chi_0^{-1}}k_\perp^4. \tag{4.10}$$

It is easy to see that these reduce to the expressions derived in the bulk case, in the limit in which membrane fluctuations are unaffected by the wall.

V. NONEQUILIBRIUM MEMBRANES: MODE STRUCTURE IN BULK

We now discuss the dynamic correlations and mode structure of an impermeable two-component membrane with active force centers, when placed far away from confining walls. For inactive force centers, the static properties of such a system are given by the free energy of Eq. (2.2). However, in the presence of nonequilibrium driving forces, thermodynamic restoring forces must be supplemented by additional terms. These are discussed briefly here.

First, we must account for nonequilibrium forces. In the context of the membrane-pump system, these arise due to the action of the pumps. However, since pumping refers to forces internal to the system, the force density exerted on the whole system (membrane + pump + solvent) must integrate to zero on length scales comparable to the pump size. A simple representation of this requirement idealizes the pump as two force centers of opposite sign but equal magnitude separated by a distance $w^\uparrow+w^\downarrow$ and embedded in the bilayer. Such a ''force dipole'' representation was introduced in Ref. [28] and will represent an important part of our discussion here.

Second, we must account for the nonequilibrium nature of the noise present in the fluid due to the switching on or off of the pumps. For simplicity, we assume that this noise is δ correlated. Such an assumption corresponds to a coarse graining in time across the typical pump ''dead'' time which separates two pumping events. The nonequilibrium active noise can now be absorbed into the correlation function of the equilibrium noise [Eq. (3.9)]. However, note that the ''temperature'' which appears in this correlator is not the thermodynamic temperature. To smoothly interpolate to the equilibrium case, we must redefine this temperature as an effective temperature, which becomes the true thermodynamic temperature in the absence of activity.

Several of the results in this section overlap with those of Manneville *et al.* (Ref. [28]), in which the effects of permeation and curvature induced activity on the dynamics of an active membrane in bulk are considered. In our model we assume impermeability from the outset and neglect the effects of curvature induced activity. Such a minimal model makes the calculation of the correlation functions of a wall-bounded active membrane more tractable and transparent.

We begin by writing the equations of motion for the various fields. The fluid is incompressible and hence obeys Eq. (3.10). Since no external forces act on the fluid+membrane +proteins system, momentum is conserved [40,41] and Eq. (3.7) holds. The momentum current tensor π_{ij} is then

$$\pi_{ij}=P\delta_{ij}+\rho v_i v_j-\eta(\nabla_i v_j+\nabla_j v_i)+S_{ij}+A_{ij}, \tag{5.1}$$

where $\mathbf{v}(\mathbf{r},t)$ is the three-dimensional fluid velocity, $P(\mathbf{r},t)$ is the three-dimensional pressure field. S_{ij} is the contribution to the stress tensor due to membrane conformations and A_{ij} is the contribution from the active pumps. Equations (3.8) and (3.10) can be solved as before for the transverse (t), longitudinal (l), and z components of the fluid velocity, in terms of the t, l, and z components of the force $\mathbf{F}(\mathbf{r},t)$ acting on the fluid due to membrane and pump stresses. The transverse components will be ignored as before; $\mathbf{f}_\mathrm{h}(\mathbf{r},t)$ represents the thermal noise present in the bulk of the fluid.

We can now calculate $v_z(\mathbf{k}_\perp,0,t)$ by solving the equations of motion with appropriate boundary conditions. The equation of motion for the height field in the absence of permeation is $\partial_t h(\mathbf{r}_\perp,t)=v_z(\mathbf{r}_\perp,0,t)$, where $v_z(\mathbf{r}_\perp,0,t)$ is the velocity of the surrounding fluid at the mean position of the membrane. The membrane does not feel the forces from the pumps directly because of the absence of permeation but only via the forces the pumps exert on the fluid [28,29]. The contribution of the bulk equilibrium thermal noise present in the fluid to the z component of the fluid velocity at $z=0$ is f_z. The statistics of f_z have been discussed earlier (see Sec. III). The density difference field obeys the diffusion equation Eq. (3.2). As in the equilibrium case, we work in the linearly stable regime.

The membrane fluctuates about $z=0$. The force exerted by the membrane and pumps on the fluid is of the form

$$\mathbf{F}(\mathbf{k}_\perp,z')=\delta(z')\left[-\frac{\delta F}{\delta h}\hat{\mathbf{z}}+F_l^m\hat{\mathbf{k}}_\perp\right]+f_{pump\text{-}fluid}\hat{\mathbf{z}}. \tag{5.2}$$

The force $f_{pump\text{-}fluid}$ is modeled as a dipolar force density with force centers located asymmetrically about the $z=0$ plane,

$$f_{pump\text{-}fluid}=f\psi(\mathbf{r}_\perp,t)[\delta(z'-w^\uparrow)-\delta(z'+w^\downarrow)], \tag{5.3}$$

where f sets the scale for the active force.

We again solve for the unknown coefficient F_l^m using membrane incompressibility. This imposes $v_l(k_\perp,0,t)=0$ in Eq. (3.11). The hydrodynamic velocity obtained at $z=0$ using Eq. (3.11) enters in the equation of motion for the height field:

$$\partial_t h(\mathbf{k}_\perp,t)=\frac{1}{4\eta k_\perp}\left[-\frac{\delta F}{\delta h}+f\psi(\mathbf{k}_\perp,t)\Omega(k_\perp,w^\uparrow,w^\downarrow)\right]+f_z(\mathbf{k}_\perp,0,t), \tag{5.4}$$

where

$$\Omega(\mathbf{k}_\perp,w^\uparrow,w^\downarrow)=-e^{-k_\perp w^\downarrow}(1+k_\perp w^\downarrow)+e^{-k_\perp w^\uparrow}(1+k_\perp w^\uparrow) \tag{5.5}$$

$$=\frac{k_\perp^2}{2}(w^{\downarrow 2}-w^{\uparrow 2})+\frac{k_\perp^3}{3}(w^{\downarrow 3}-w^{\uparrow 3})+\cdots. \tag{5.6}$$

Note that $\Omega(k_\perp,w^\uparrow,w^\downarrow)$ vanishes when $w^\uparrow=w^\downarrow$, implying that an asymmetry in the position of the force centers above and below the membrane midplane is needed for a nonzero active (nonequilibrium) force. The equations of motion are

$$\partial_t\begin{pmatrix}h(\mathbf{k}_\perp,\omega)\\ \psi(\mathbf{k}_\perp,\omega)\end{pmatrix}=-\begin{pmatrix}\dfrac{\sigma k_\perp^2+\kappa k_\perp^4}{4\eta k_\perp} & \dfrac{\kappa\bar{H}k_\perp^2-f\Omega(k_\perp,w^\uparrow,w^\downarrow)}{4\eta k_\perp}\\ \Lambda\kappa\bar{H}k_\perp^4 & \Lambda\chi_0^{-1}k_\perp^2\end{pmatrix}\times\begin{pmatrix}h(\mathbf{k}_\perp,\omega)\\ \psi(\mathbf{k}_\perp,\omega)\end{pmatrix}+\begin{pmatrix}f_z(\mathbf{k}_\perp,\omega)\\ i\mathbf{k}_\perp\cdot\mathbf{f}_\psi\end{pmatrix}. \tag{5.7}$$

The correlation function for the height fluctuations of an active membrane (about the flat, stable, steady-state phase) is then

$$\langle h^*(\mathbf{k}_\perp,t)h(\mathbf{k}_\perp,t)\rangle=\frac{k_BT}{4\eta k_\perp}\times\frac{\left(\tau_\psi^{-1}+\tau_h^{-1}-\tau_a^{-1}+\dfrac{f^2\Omega^2}{4\eta k_\perp\chi_0^{-1}}\right)}{(\tau_\psi^{-1}+\tau_h^{-1})(\tau_h^{-1e}+\tau_a^{-1})}, \tag{5.8}$$

where the quantities $\tau_\psi^{-1},\tau_h^{-1}$ have been defined earlier. τ_h^{-1e} has the same form as τ_h^{-1} with κ replaced by κ_e. The relaxation time

$$\tau_a^{-1}=\frac{1}{4\eta k_\perp}\frac{\kappa\bar{H}}{\chi_0^{-1}}f\Omega(k_\perp,w^\uparrow,w^\downarrow)k_\perp^2. \tag{5.9}$$

We discuss the positivity of the right-hand side of Eq. (5.8) after discussing the mode structure of the system. From the equal time correlation function we can calculate the quantity $\langle h^2(L)\rangle$. To lowest order we find

$$\langle h^2(L)\rangle=k_BTL^2\Big/\left[\kappa_e-\frac{\kappa\bar{H}\mathcal{P}w}{\chi_0^{-1}}\right] \tag{5.10}$$

for tensionless membranes and

$$\langle h^2(L)\rangle=\frac{k_BT}{\sigma}\ln(L/a) \tag{5.11}$$

for tense membranes. $\mathcal{P}=f(w^{\uparrow 2}-w^{\downarrow 2})/2w$, where w is the size of the pump, represents the force dipole energy. It is estimated to be a few k_BT [28]. These results imply that on large length scales, pumping activity does not change the roughness of equilibrium, tensionless membranes but shifts the *temperature* at lowest order to

$$T^{eff}=T\left[1+\frac{\kappa\bar{H}\mathcal{P}w}{\kappa_e\chi_0^{-1}}\right]. \tag{5.12}$$

For tense membranes, both roughness and effective temperature are not affected by nonequilibrium effects at large length scales since the divergence of the height correlation function with system size remains logarithmic.

In the stable regime Eq. (5.10) is positive, as discussed later in this section. The correlation function in Eq. (5.10) is calculated from the lowest-order diffusive terms. Manneville *et al.* [28] calculate an identical correlation function but *neglect* these diffusive terms in favor of higher-order terms on the grounds that the diffusion constant is small in the physical situation. For this reason the effective temperature we calculate in Eq. (5.12) *is not* the effective temperature that Manneville *et al.* calculate and compare with experiment.

The height correlation functions we calculate in our linearized approach [Eqs. (5.10) and (5.11)] show that the roughness of an active fluid membrane is unchanged in the bulk. The calculation of Ref. [29] obtains an enhanced roughening. The reason is the different nature of the force densities used in both the calculations. Prost *et al.* consider a model in which the membrane feels the activity of the pump *only* as a consequence of permeation. In our calculation, Eq. (5.4) written as a Langevin equation is

$$\partial_t h(\mathbf{k}_\perp,t)+\tau_h^{-1}h(\mathbf{k}_\perp,t)=-\frac{(\kappa\bar{H}k_\perp^2-f\Omega)}{4\eta k_\perp}\psi+f_z. \tag{5.13}$$

We assume that the pump is always in the active state over the time scales of our interest. The forces on the right-hand side of the above equation arise from three sources: an elastic part (due to the curvature-concentration coupling term in the free energy), an asymmetric dipole part (from the activity of the pumps), and the thermal noise from the fluid. Since the membrane is impermeable it does not feel the active forces or thermal noise directly but only via the fluid. We emphasize that in the present model the membrane senses active forces and thermal noise even when $\lambda_p=0$, in contrast to the model of Prost *et al.* [29].

The modes are

$$\omega_1,\omega_2=\frac{1}{8\eta}(\sigma k_\perp+4\eta D k_\perp^2+\kappa k_\perp^3)\pm\frac{1}{8\eta}[\sqrt{(\sigma k_\perp+4\eta D k_\perp^2+\kappa k_\perp^3)^2-16\eta(D\kappa_e k_\perp^5+D\sigma k_\perp^3+f\bar{H}\kappa\Lambda\Omega k_\perp^3)}].$$

In the absence of curvature-concentration coupling, nonequilibrium effects vanish to lowest order and the modes are those for an equilibrium membrane with hydrodynamic interactions $\omega_1,\omega_2=(\sigma k_\perp+\kappa k_\perp^3)/4\eta, Dk_\perp^2$. When $\sigma=0$, curvature-concentration coupling modifies this mode structure to

$$\omega_1\approx\frac{\kappa_e}{4\eta}k_\perp^3+\frac{f\kappa\bar{H}(w^{\downarrow 2}-w^{\uparrow 2})}{8\eta\chi_0^{-1}}k_\perp^3,$$

$$\omega_2\approx Dk_\perp^2+\frac{\kappa\bar{H}[2\kappa\bar{H}-f(w^{\downarrow 2}-w^{\uparrow 2})]k_\perp^3}{8\eta\chi_0^{-1}}. \quad (5.14)$$

We see that to lowest order, nonequilibrium effects are apparent only at nonzero curvature-concentration coupling. We also observe that the nonequilibrium terms vanish when $w^\downarrow=w^\uparrow$.

For stability we require that all the eigenmodes of the system must decay. This means that ω_1 and ω_2 are positive. To the order above, this imposes $\kappa_e/\kappa-1<\bar{H}\mathcal{P}w/\chi_0^{-1}<\kappa_e/\kappa$. If $\bar{H}\mathcal{P}$ is such that this condition is satisfied, then the system is stable independent of the sign of this parameter. Typical experimental values such as $\kappa=10k_BT$, $\kappa\bar{H}=wk_BT$, $\chi_0^{-1}=w^2k_BT$, $f=10^{-12}$ N satisfy the above conditions [44].

VI. NONEQUILIBRIUM MEMBRANES: MODE STRUCTURE IN THE PRESENCE OF A WALL

We now study the effect of a wall located at $z=0$ on the fluctuations of an active membrane located at $z=d$. The forces exerted on the fluid by the membrane, pumps, and the wall are now given by

$$\mathbf{F}(\mathbf{k}_\perp,z')=\delta(z'-d)\left[-\frac{\delta F}{\delta h}\hat{\mathbf{z}}+F_l^m\hat{\mathbf{k}}_\perp\right]+f\psi(\mathbf{r}_\perp,t)\times[\delta(z'-d-w^\uparrow)-\delta(z'-d+w^\downarrow)]\hat{\mathbf{z}}+\delta(z')\times[F_z^w\hat{\mathbf{z}}+F_l^w\hat{\mathbf{k}}_\perp]. \quad (6.1)$$

We need to determine the unknown coefficients F_l^m, F_z^w, and F_l^w. The following three boundary conditions are used, following Eqs. (3.11):

$$v_z(\mathbf{k}_\perp,0,t)=v_l(\mathbf{k}_\perp,0,t)=v_l(\mathbf{k}_\perp,d,t)=0. \quad (6.2)$$

These boundary conditions impose the following constraints: (i) $v_z(\mathbf{k}_\perp,z=0)=0$, i.e., the z component of the velocity vanishes at the wall, (ii) $v_l(\mathbf{k}_\perp,z=0)=0$, i.e., the l component of the velocity vanishes at the wall, and (iii) $v_l(\mathbf{k}_\perp,z=d)=0$, i.e., the membrane located at $z=d$ is incompressible. We neglect the tangential component as before.

The values of the three unknown coefficients are obtained on solving the three simultaneous equations [Eq. (3.11)] together with the three boundary conditions. These values can then be used to obtain the velocity of the fluid at the location of the membrane. The velocity thus obtained has a part arising from equilibrium elastic forces and a part arising from nonequilibrium active forces. These can be separated out, yielding the equation of motion for the height field as

$$\partial_t h(\mathbf{k}_\perp,t)=v_z(\mathbf{k}_\perp,d,t)=-\mathcal{G}\left(\frac{\delta F}{\delta h}\right)+\mathcal{G}^{aw}(f\psi)+\tilde{f}_z(\mathbf{k}_\perp,t), \quad (6.3)$$

where $\mathcal{G}$ is the usual hydrodynamic kernel obtained for the equilibrium fluctuations in the presence of a wall, and the new *nonequilibrium* term

$$\mathcal{G}^{aw}(k_\perp,d,w^\uparrow,w^\downarrow)\sim\begin{cases}\dfrac{k_\perp^2}{12\eta}(3dw^{\downarrow 2}-2w^{\downarrow 3})+O(k_\perp^4)+\cdots & \text{if } k_\perp\ll 1/d\\ \dfrac{1}{4\eta k_\perp}\Omega(k_\perp,w^\uparrow,w^\downarrow) & \text{if } k_\perp\gg 1/d\end{cases} \quad (6.4)$$

is the hydrodynamic kernel for nonequilibrium fluctuations in the presence of the wall [29] (the expanded form of this kernel is given in Ref. [45]; see also Ref. [46]). In the expressions for $\mathcal{G}$ and $\mathcal{G}^{aw}$, the lower cutoff on $k_\perp$ is the inverse of the collision length L_c, which we will calculate towards the end of this section. The membrane-wall distances we use in our calculations are such that $d\gg w^\uparrow, w^\downarrow$ and hence $w^{\downarrow 3}$ can be neglected in comparison to $dw^{\downarrow 2}$. In the long wavelength limit, $\mathcal{G}^{aw}$ depends on $w^\downarrow$ and not on $(w^\downarrow-w^\uparrow)$ as in the bulk. When the fluctuations are not affected by the wall, $\mathcal{G}^{aw}$ vanishes when $w^\uparrow$ equals $w^\downarrow$ since in bulk, the directions up and ''down'' are equivalent. The nonequilibrium contribution to the force density therefore vanishes. Thus, in this large membrane-wall distance limit, we recover

the previous situation of active membrane fluctuations where the asymmetry was crucial for nonzero active force density. The wall breaks the symmetry in the up and down directions and hence the hydrodynamic kernel picks out a particular direction and does not depend on $(w^{\downarrow}-w^{\uparrow})$ when membrane fluctuations feel the wall. The statistics for the noise $\tilde{f}_z$ in the presence of the wall follows from Eqs. (4.8).

The equal time correlation function can be calculated from

$$\partial_t \begin{pmatrix} h(\mathbf{k}_\perp,\omega) \\ \psi(\mathbf{k}_\perp,\omega) \end{pmatrix} = -\begin{pmatrix} \mathcal{G}(\sigma k_\perp^2+\kappa k_\perp^4) & \mathcal{G}\kappa\bar{H}k_\perp^2-\mathcal{G}^{aw}f \\ \Lambda\kappa\bar{H}k_\perp^4 & \Lambda\chi_0^{-1}k_\perp^2 \end{pmatrix} \times \begin{pmatrix} h(\mathbf{k}_\perp,\omega) \\ \psi(\mathbf{k}_\perp,\omega) \end{pmatrix} + \begin{pmatrix} \tilde{f}_z(\mathbf{k}_\perp,\omega) \\ i\mathbf{k}_\perp\cdot\mathbf{f}_\psi \end{pmatrix}. \tag{6.5}$$

The correlation function is

$$\langle h^*(\mathbf{k}_\perp)h(\mathbf{k}_\perp)\rangle = k_BT\mathcal{G} \times \left(\frac{\left(\tau_\psi^{-1}+\tau_h^{-1w}-\tau_a^{-1w}+\dfrac{f^2\mathcal{G}^{aw2}}{\chi_0^{-1}\mathcal{G}} \right)}{(\tau_\psi^{-1}+\tau_h^{-1w})(\tau_h^{-1w,e}+\tau_a^{-1w})} \right), \tag{6.6}$$

where

$$\tau_h^{-1w}=\mathcal{G}(\sigma k_\perp^2+\kappa k_\perp^4), \quad \tau_a^{-1w}=\mathcal{G}^{aw}\frac{\kappa\bar{H}}{\chi_0^{-1}}fk_\perp^2. \tag{6.7}$$

$\tau_h^{-1w,e}$ has the same form as τ_h^{-1w} with κ replaced by κ_e. Inserting physically reasonable values for the parameters which enter these definitions (see below), we verify that the right-hand side of the correlation function is always positive. Thus, this correlation function is calculated in a regime in which the active membrane is in a *stable* steady-state phase.

At large length scales and in the presence of a confining wall $(1/L_c \ll k_\perp \ll 1/d)$,

$$\langle h^2(L)\rangle = \frac{k_BT}{\kappa\bar{H}}\left[\frac{\epsilon'\chi_0^{-1}}{\epsilon f}+\frac{f\epsilon}{D}\right]\ln(L/a) \sim \frac{k_BT}{\Sigma}\ln(L/a) \tag{6.8}$$

for *tensionless* membranes and

$$\langle h^2(L)\rangle = k_BT\left[\frac{\epsilon'+(f^2\epsilon^2/D\chi_0^{-1})}{\epsilon'\sigma+(\epsilon f\kappa\bar{H}/\chi_0^{-1})}\right]\ln(L/a) \sim \frac{k_BT}{\sigma^e}\ln(L/a) \tag{6.9}$$

for tense membranes. We have defined the quantities $\epsilon = dw^{\downarrow 2}/(4\eta)$ and $\epsilon' = d^3/(12\eta)$ for notational convenience. Thus, the presence of the wall makes tensionless active membranes less rough than they are in bulk since the height correlation function scales now logarithmically with L instead of quadratically. Note that this effect is *opposite* to that predicted via the scaling arguments of Prost *et al.* in Ref. [29].

The effective surface tension is then

$$\Sigma = \frac{\kappa\bar{H}}{\left[\dfrac{\epsilon'\chi_0^{-1}}{\epsilon f}+\dfrac{f\epsilon}{D}\right]}. \tag{6.10}$$

This is an explicitly nonequilibrium effect, since $\Sigma=0$ when $f=0$. The induced surface tension also crucially depends on the curvature-concentration coupling $\bar{H}$ since it vanishes on setting this coupling to zero. Therefore, both nonequilibrium forces and curvature-concentration coupling are essential for the stiffening of a tensionless, active membrane near a wall.

To calculate the collision length L_c we set $h^2(L_c)\approx d^2$ in Eqs. (6.8) and (6.9). Thus

$$L_c(d)=\begin{cases} a\exp(\Sigma d^2/k_BT) & \text{if } \sigma=0 \\ a\exp(\sigma^e d^2/k_BT) & \text{if } \sigma\neq 0. \end{cases} \tag{6.11}$$

The eigenmodes have frequencies

$$\omega_1,\omega_2=\frac{1}{2}(Dk_\perp^2+\mathcal{G}\sigma k_\perp^2+\mathcal{G}\kappa k_\perp^4)\pm\frac{1}{2}[\sqrt{(Dk_\perp^2+\mathcal{G}\sigma k_\perp^2+\mathcal{G}\kappa k_\perp^4)^2-4(f\mathcal{G}^{aw}\bar{H}\kappa\Lambda k_\perp^4+\mathcal{G}D\kappa_e k_\perp^6+\mathcal{G}D\sigma k_\perp^4)}].$$

In the absence of curvature-concentration coupling, nonequilibrium forces have no effect on the modes and the modes reduce to those of an equilibrium membrane in the presence of a wall, within the linearized theory. For tensionless membranes with nonzero curvature-concentration coupling, nonequilibrium effects become apparent in the mode structure. The modes to lowest order are

$$\omega_1 \approx \frac{f\mathcal{G}^{aw}\kappa\bar{H}}{\chi_0^{-1}}k_\perp^2+\mathcal{G}\kappa_e k_\perp^4,$$

$$\omega_2 \approx Dk_\perp^2+\frac{\kappa\bar{H}(\kappa\bar{H}\mathcal{G}k_\perp^2-f\mathcal{G}^{aw})}{\chi_0^{-1}}k_\perp^2. \tag{6.12}$$

When membrane fluctuations are influenced by the wall, $\mathcal{G}=d^3k_\perp^2/12\eta$ and $\mathcal{G}^{aw}=dw^{\downarrow 2}k_\perp^2/4\eta$ to lowest order. Far away from the wall the modes reduce to those of a nonequilibrium membrane in the bulk.

It may seem that for initially tensionless membranes near a wall, the induced surface tension Σ can have either sign

depending on the sign of the curvature-concentration coupling $\bar{H}$. We will show that for stability, it is imperative that $\bar{H}$ is positive for active membranes close to walls; a negative $\bar{H}$ leads to a negative ω_1, and thus an instability. The requirement of positivity of $\bar{H}$ in the stable regime ensures that our approximations of considering fluctuations at linear order about an initial flat steady-state phase are self-consistent.

We now present a physical argument for why pumping activity should lead to the *stiffening* of active, tensionless membranes near repulsive walls. The calculations of Sec. IV indicate that the eigenvalues governing the relaxation of a tense, equilibrium membrane near a wall are $\sigma d^3 k_\perp^4/(12\eta)$ and $Dk_\perp^2$. For an active, tensionless membrane near a wall these eigenvalues are $\kappa \bar{H} f d w^{\downarrow 2} k_\perp^4/4\eta\chi_0^{-1}$ and $Dk_\perp^2$. The coefficient of the slower mode for the active case $[O(k_\perp^4)]$ can be written as $(\kappa \bar{H}\epsilon f/\epsilon' \chi_0^{-1}) d^3/(12\eta)$ and compared to the expression for the slowest relaxation mode for tense, equilibrium membranes. This comparison suggests that an apparent surface tension $(\kappa \bar{H}\epsilon f/\epsilon' \chi_0^{-1})$ is induced due to purely nonequilibrium effects, even for a membrane with vanishing bare surface tension.

In the linearized theory, the induced surface tension obtained using the above argument and the surface tension Σ obtained from the coefficient of the logarithmic term in the calculation of the height roughness coincide. It is thus apparent that the presence of the wall smoothens a tensionless, active membrane by making it relax faster than its tensionless equilibrium counterpart [whose slowest mode is $O(k_\perp^6)$].

Why does an active membrane relax faster near the wall? Height field fluctuations are predominantly governed by the ψ field [the second term in Eq. (6.3)] because this term is $O(k_\perp^2)$ in contrast to the other terms which are of higher order in wave number. Thus, this equation to lowest order in wave number is $\partial_t h = \mathcal{G}^{aw} f \psi$. Since the ψ field relaxation is fast compared to the height field, we may assume that, over the larger time scales of interest to us in studying the dynamics of the height field, ψ relaxes instantaneously to $\psi_s = -(\kappa \bar{H}/\chi_0^{-1}) k_\perp^2 h$. The effective height field equation now becomes

$$\partial_t h = -\frac{f\epsilon\kappa\bar{H}}{\chi_o^{-1}} k_\perp^4 h. \tag{6.13}$$

Note that the coefficient of the $k_\perp^4$ term in the above equation can be rewritten to obtain the induced surface tension Σ, as explained in the previous paragraph.

The above argument invokes and crucially requires the presence of the wall, activity, and curvature-concentration coupling. In physical terms, height field fluctuations are accompanied by fast relaxation of the protein density field ψ. This relaxation is to a steady-state value determined by the pump diffusion coefficient and curvature-concentration coupling. The pumping activity is then inhomogeneously distributed along the membrane surface. To see the physical consequences of this inhomogeneous distribution, consider a fluctuation creating a region of positive curvature. If $\bar{H}$ is positive, the number of up pumps in the region of positive curvature tends to decrease with time and the decrease in the pumping activity causes a smoothening of the membrane. However, if $\bar{H}$ is negative, the region of positive curvature attracts more up pumps, which further bootstraps the height fluctuation, leading to an instability.

The explicit expressions obtained above also permit us to estimate the collision length for tensionless active membranes in the presence of repulsive walls. Using estimates given in Ref. [28] for various physical quantities ($\kappa = 10^{-20}$ J, $\bar{H} = 5$ nm, $f = 10^{-12}$ N, d=1 μm, $\chi_0^{-1} = 10^{-21}$ J m^2), we find that the induced surface tension $\Sigma \sim 10^{-7}$–10^{-8} N/m. This translates to collision lengths L_c of the order of 10^5–10^6 times molecular scales. Assuming molecular length scales of the order of 1 nm, this calculation would then predict collision lengths of the order of millimeters, for membranes with physical parameters in the above range.

VII. CONCLUSIONS

This paper has presented a calculation of the linearized hydrodynamics of two-component membranes including the effects of curvature-concentration coupling, specifically in the context of a simple model for a biological membrane with protein and lipid constituents. We have studied in some detail the case in which such membranes are placed close to as well as far away from a confining repulsive wall. We have considered the case in which such proteins are ''active,'' in which case they exert nonequilibrium forces on the solvent, as well as the case of ''inactive'' pumps, corresponding to thermal equilibrium.

The calculations we report are for a model that shares many similarities with that of Ref. [28]. Manneville *et al.* consider the more general case of a membrane with a finite permeability whereas we work with an impermeable membrane throughout. In addition, we neglect the possible effects of curvature on activity. However, the relatively simpler structure of our model enables us to establish a connection with equilibrium results in a more transparent fashion. We work in a framework in which boundary conditions are explicitly implemented. This is especially of use when we consider the linearized theory of fluctuations of an active membrane near a wall. To the best of our knowledge, the results we report for the linearized hydrodynamics of active and inactive two-component membranes near walls are new, as are our results for the equilibrium case far away from confining walls.

We find, far away from the walls and consistent with the results of Manneville *et al.* [28], that the presence of active pumps on the membrane does not change its roughness. In the case of tensionless membranes, fluctuations can be described in terms of an effective temperature, which is shifted from the thermodynamic temperature by nonequilibrium effects due to pumping. These terms depend on the quantity $(w^{\downarrow} - w^{\uparrow})$, a measure of the degree of asymmetry in the positions of the force centers. We reiterate the observation of Manneville *et al.* in Ref. [28] that this asymmetry is essential

for nonequilibrium effects to be visible in the membrane dynamics.

We find that the activity of the pumps within the *stable* regime appears to *smoothen* out height fluctuations in the presence of the wall since height correlations scale only logarithmically with L for both tense and tensionless membranes. This smoothening out within the linear theory presumably competes with the possible enhancement of fluctuations, if any, which might arise in a theory that incorporates the effects of nonlinearities. We have provided a physical interpretation of this effect in this paper and obtained estimates for the appropriate collision length. We find that this collision length can be significantly larger than typical length scales for biological vesicles, indicating the relevance of the physical effects we study here to a complete description of the hydrodynamics of active pump-membrane systems. It is important to note that this smoothening relies crucially on the existence of curvature-concentration coupling terms, neglected in previous work on the near-wall case. These results suggest that earlier ones based on scaling arguments which predicted an *enhancement* in the roughness may need to be reevaluated in the light of this work.

We have also calculated the crossover function $\mathcal{G}^{aw}$ and linearized mode structure in the presence of the wall. While the positional asymmetry in the placement of force centers was crucial for active membrane fluctuations in bulk, we point out that nonequilibrium effects are apparent even for symmetrically placed pumps in the presence of a wall. This unusual, although not totally unexpected, result does not appear to have been noticed before.

We also find that the requirement of stability for active membranes near a wall constrains $\bar{H}$, the coefficient coupling concentration with curvature, to be positive. This can have interesting consequences. Consider two active membranes, one with positive $\bar{H}$ and the other with negative $\bar{H}$, fluctuating in the bulk. Provided $\bar{H}\mathcal{P}$ for each lies in the window of stability in the bulk, these membranes are stable. Now bring both close to a wall. The membrane with positive $\bar{H}$ will remain stable. The fluctuations of the membrane with negative $\bar{H}$, however, will turn unstable; its final fate will be decided by the nonlinear terms we have omitted from our discussion.

Because our calculation is an explicitly *linear* one, we cannot address issues such as the relevance of such nonlinear terms. The arguments of Prost, Manneville, and Bruinsma, which predict an anomalous roughening of active membranes near walls, rely on identifying the role of such nonlinear terms and estimations of their importance via scaling arguments. It would be interesting to construct similar arguments for the model studied here, particularly since our results in the linear case indicate that fluctuations of an active membrane near a repulsive wall can be *smoothened* in some regimes, rather than roughened relative to the equilibrium case. Further work along these lines is currently in progress.

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APPENDIX

1. A note on the impermeability of the membrane

In this section we present an argument, following Manneville *et al.*, to justify the assumption of membrane impermeability. Had we incorporated permeation due to activity in our calculations, active-permeative and active-hydrodynamic terms would appear as $[\lambda_p - k_\perp(w^{\uparrow 2} - w^{\downarrow 2})/8\eta] f\psi$ (λ_p is the permeation coefficient) [28]. The crossover length scale from the hydrodynamic to the permeative regime occurs when $k_\perp = 8\eta\lambda_p/w(w^\uparrow - w^\downarrow)$, i.e., $l_\perp = \pi w(w^\uparrow - w^\downarrow)/4\eta\lambda_p$ (w is the size of the pump). With $\lambda_p = 10^{-12}$ m^3/N s, $\eta = 10^{-3}$ kg/m s, and $w = 5\times 10^{-9}$ m [28], this crossover length scale is estimated to be of the order of 1 cm. The length scales of concern to us are in the micron and submicron range [28]. Thus, at these length scales, effects due to permeation can be ignored. These arguments can be extended to estimating the role of passive permeative vs hydrodynamic modes of relaxation; we conclude that at the length scales of interest to us, the assumption of membrane impermeability remains valid.

In the presence of a wall, active-permeative terms would dominate if $\lambda_p > \mathcal{G}^{aw}$. This means that the length scale above which permeation dominates is $\sqrt{\pi d w^{\downarrow 2}/2\eta\lambda_p}$. This length is ≈ 40 μm when the membrane-wall distance $d = 1$ μm. Hence, in the regimes of interest to us, permeation can be ignored and relaxation is purely hydrodynamic. Hence, we assume $\lambda_p = 0$ to derive our results.

2. A modified model for active membranes

We present here results for a model in which proteins are located asymmetrically across the bilayer with their centers of mass displaced above or below the bilayer midpoint (see Fig. 3). The head and tail of the pumps are then located at distances w_l and w_s, respectively, from the midpoint of the bilayer. Note that in this model, the up and down pumps are related to each other by reflection along the $z = 0$ line unlike the model of Fig. 2. This model can be used to describe situations in which proteins are attached to planar bilayer membranes which are symmetric on both sides, unlike vesicles in which the inner leaf and outer leaf need not be symmetrically preferred by the protein.

When pumps are active on such a membrane in bulk, the force density due to the pumps acting on the fluid has the form

$$f_{pump\text{-}fluid} = f[\delta(z - w_l)n^\uparrow - \delta(z + w_s)n^\uparrow - \delta(z + w_l)n^\downarrow + \delta(z - w_s)n^\downarrow], \tag{A1}$$

where $n^\uparrow$ and $n^\downarrow$ are the local number of up and down pumps, respectively. On repeating the calculations done in Sec. V, we find that the z component of the hydrodynamic velocity has the same form as shown in Eq. (5.7) with $\Omega(k_\perp, w^\uparrow, w^\downarrow)$ replaced by $\Omega'(k_\perp, w_l, w_s) = -e^{k_\perp w_s}(1$

$+k_\perp w_s)+e^{k_\perp w_l}(1+k_\perp w_l)$, which vanishes when $w_l=w_s$ (i.e., for symmetrically placed proteins). Correlation functions and relaxation times can be read off from the corresponding expressions in Sec. V by identifying $w^\downarrow$ and $w^\uparrow$ with w_s and w_l, respectively. For example,

$$\langle h^*(\mathbf{k}_\perp,t)h(\mathbf{k}_\perp,t)\rangle = \frac{k_BT}{4\eta k_\perp}\left(\frac{\left(\tau_\psi^{-1}+\tau_h^{-1}-\tau_a^{-1}+\dfrac{f^2\Omega^2}{4\eta k_\perp \chi_0^{-1}}\right)}{(\tau_\psi^{-1}+\tau_h^{-1})(\tau_h^{-1e}+\tau_a^{-1})}\right), \tag{A2}$$

where the quantities $\tau_\psi^{-1},\tau_h^{-1}$ have been defined earlier. τ_h^{-1e} has the same form as τ_h^{-1} with κ replaced by κ_e. The relaxation time

$$\tau_a^{-1}=\frac{1}{4\eta k_\perp}\frac{\kappa\bar{H}}{\chi_0^{-1}}f\Omega(k_\perp,w_l,w_s)k_\perp^2. \tag{A3}$$

We note here that this identification is purely notational since $(w^\downarrow,w^\uparrow)$ and (w_s,w_l) physically represent different lengths in the two models.

When the calculations are repeated for such a model in the presence of a repulsive wall, we find that the hydrodynamic velocity acquires an additional contribution from the ϕ (total protein density) field. In this linearized calculation we assume that the compressibility associated with the ϕ field is very large and ignore its dynamics. However, we do not expect this contribution to alter the long wavelength fluctuations of the active membrane near the wall significantly.

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[44] The right-hand side of Eq. (5.8) must be positive. Since other quantities are explicitly positive, we require that $\tau_h^{-1}-\tau_a^{-1}>0$, implying $\widetilde{H\mathcal{P}}w/\chi_0^{-1}\geqslant -1$. We see that the values of $\widetilde{H\mathcal{P}}$ satisfying the stability requirement automatically satisfy the condition of the positivity of the correlation function. We thus conclude that the correlation function becoming negative can be associated with the onset of an instability in the system.

[45] The complete expression for the nonequilibrium kernel is

$$\mathcal{G}^{aw}(k_\perp,d,w^\uparrow,w^\downarrow)=\frac{1}{4\eta k_\perp(-1+e^{2k_\perp d}+2k_\perp d-2k_\perp^2 d^2)}\times[-2\cosh(2k_\perp d-k_\perp w^\downarrow)+k_\perp w^\downarrow 2\sinh(k_\perp w^\downarrow)+2(1+2k_\perp^2 d^2-2k_\perp^2 dw^\downarrow)\cosh(k_\perp w^\downarrow)+2(2k_\perp d-k_\perp w^\downarrow)\sinh(k_\perp w^\downarrow)+2e^{-k_\perp w^\uparrow}(1+k_\perp w^\uparrow)\times\cosh(2dk_\perp)+2(1+2k_\perp^2 d^2)e^{-k_\perp w^\uparrow}].$$

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